EULER DECONVOLUTION OF POTENTIAL FIELD MAGNETOMETER AND ITS USE IN DELINEATION OF REGIONAL FAULTS WITH DATA FROM NORTHERN PERU

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ABSTRACT:
The geomagnetic field measurements can be used to determine the structure of the earth, because the rocks often contain magnetic minerals. The interpretation of data of this nature will determine in some cases, geological characteristics that would help contribute to the success of mining or oil exploration. The methodology to calculate the depth and location of magnetic sources (dam and failure), through the Euler Deconvolution method and application is made for three magnetic profiles extracted from a data grid in northern Peru. It then lists a series of conclusions about the potential of this methodology.

1 INTRODUCTION

Traditional methods in interpreting gravitational magnetometer require, in addition to experts in the field, knowledge to perform geological and geophysical modeling (Nabighian et al. 2005).

There are other methods, called semi-empirical, requiring no other knowledge or other specialists. These methods are applied directly to the data, gridded and mapped. They are particularly successful in detecting contacts, with differences in susceptibility, structures similar to dikes, spheres, horizontal cylinders, vertical pipes, etc.

One method widely used to estimate the depth and location of the sources that cause the magnetic anomalies, is the method of Euler Deconvolution.

2 EULER DECONVOLUTION FOR MAGNETIC DAM AND FAILURE

Thompson in 1982 proposed a technique for analyzing magnetic profiles based on Euler's relationship for homogeneous functions. The Euler deconvolution technique uses first-order derivatives of x, y, and z to determine the location and depth of various idealized sources that produce them. Among the most common are the sphere, cylinder, thin dike and the failure, the latter two being the case studied in this paper. Each of these sources are characterized by an structural index, the method is in principle applicable for all types of bodies.

Reid (1990) extended the technique to 3D data by applying the Euler operator data windows on a grid. Silva and Barbosa (2003) and Introcaso et al. (2008) among others helped in the understanding and applicability of the technique.

2.1 Theory

Consider any function of three Cartesian coordinates x, y, and z denoted by f (x, y, z). The observation plane is taken as z = 0 and positive down and based on that choose the direction of x is north and y is east.
2.1.1 Euler equation

The function \( f(x, y, z) \), is said to be homogeneous of degree \( n \) if:

\[
f(tx, ty, tz) = t^n f(x, y, z)
\]  

(2.1)

In addition, we show that if \( f(x, y, z) \) is homogeneous of degree \( n \), the following equation is satisfied.

\[
x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = nf
\]  

(2.2)

This partial differential equation is known as Euler’s equation of homogeneity or simply Euler’s equation. Suppose that \( f(x, y, z) \) has the following general form:

\[
f(x, y, z) = \frac{G}{r^N}
\]  

(2.3)

where \( r^2 = x^2 + y^2 + z^2 \)

\( N = 1, 1.5, 2, 2.5, 3,... \)

\( G: \) constant independent of \( x, y, z \)

Equation 2.3 is clearly homogeneous of degree \( n = -N \). Many single point magnetic sources are in the form of this equation.

2.1.2 The structural index

Consider a point source (point mass, magnetic dipole, etc) located at the point \( x_0, y_0, z_0 \) on the measurement plane. The total magnetic intensity is of the form:

\[
\Delta F(x, y) = f(x-x_0), (y-y_0), z_0)
\]  

(2.4)

Reid et al. (1990) proposed the relation:

\[
(x - x_0) \frac{\partial \Delta F}{\partial x} + (y - y_0) \frac{\partial \Delta F}{\partial y} - z_0 \frac{\partial \Delta F}{\partial z} = -N \Delta F(x, y)
\]  

(2.5)

The gradients in the three Cartesian directions can be calculated using standard potential theory in the domain of space or wave number. In some cases the vertical gradient could have been measured and used directly in Equation 2.5. This equation could be used to analyze mapped magnetic data (Cook 1950).

Let us assume that the transverse gradient, the second term of equation 2.5, is zero. This would be a 2D anomaly in which the equation reduces to the expression:

\[
(x - x_0) \frac{\partial \Delta F}{\partial x} - z_0 \frac{\partial \Delta F}{\partial z} = -N \Delta F(x, y)
\]  

(2.6)

The derivatives or gradients in Equation 2.6 can be measured or more commonly calculated from the data, and it would therefore only be sufficient to know the \( x_0, z_0 \) and \( N \). The coordinates \( (x_0, z_0) \) represent the depth and location along the profile of an equivalent point source and \( N \) correspond to the kind of font that best represents the type of anomaly. Several simple models have prescribed values of \( N \) (Smellie 1956).

Many geological features have structural indices that best describe its depth and location, so much so that for example a thin 2D dam has a structural index \( N = 1 \) at the magnetic pole, while a contact has a structural index lower than 0.5, as we checked with this method.

Equation 2.6 can be solved exactly for the unknown values \( x, z \) and \( N \) evaluating the derivatives and the total field values in the three different coordinates along the profile. This result in three linear equations and three unknowns which can, in principle, be solved if the determinant of coefficients is not zero.

The implementation of Equation 2.6 on directly observable data is not useful for three reasons:

- Many anomalies, even at the magnetic pole, prefer high structural indices, i.e., they are more bipolar in nature. However, the low structural indices are better estimates of depth (Thompson 1982).
The absolute level of the field is rarely known. The regional fields due to local anomalies are almost always present.

On real data, the anomalies are rarely represented exactly by point sources.

These factors make the exact solution of Equation 2.6 very suspicious and erratic. To overcome these problems, some methods have been developed that will be described below:

The problem of forcing the method to lower depths for structural indices is determined by an analysis for a number of prescribed structural indices. Then one checks which one produces better solutions by observing the cluster of solutions located in a region of space.

The problem of removing the "bias" of the observed magnetic field is to solve it by assuming that the anomalous field is disturbed by a constant amount B in a window in which Equation 2.6 is being evaluated. The observed number is:

\[ F(x) = \Delta F(x) + B \]  

(2.7)

where B is constant in the x coordinate of the portion of the profile where the analysis is being performed. Finally solving equation 2.7 for \( \Delta F \) and substituting in equation 2.6 and arranging the terms give:

\[ x_o \frac{\partial \Delta F}{\partial x} + z_o \frac{\partial \Delta F}{\partial z} + NB = x \frac{\partial \Delta F}{\partial x} + NF \]  

(2.8)

Since the real anomalies are only approximations of the simple models, the third main problem is solved by creating an overdetermined set of linear equations. If one evaluates the equation 2.8 in four or more points within a window of a profile, this results in an overdetermined set of linear equations. For this analysis, we used a total of 7 points, i.e. the creation of a total of seven equations for three unknown variables \( x_0 \), \( z_0 \), and B, which will be solved by using the method of least squares.

1.1.1 Application of Euler deconvolution to the case of a dam and a failure

The solution by the method of least squares of an overdetermined set of equations also produces estimates of the standard deviation of the parameter \( z_0 \), \( \sigma \). This amount is treated as an error bar on the estimated depths and forms the basis for an algorithm that determines whether a solution is accepted or not.

The magnetic anomaly caused by a magnetic body can be exactly duplicated by placing a suitable distribution of magnetic poles on the surface of the body that causes it. A thin magnetic intrusive body would have induced positive pole on its upper surface and negative poles induced in the subsurface. In the distance, such magnetic sources have bipolar features. On the other hand, the magnetic bodies are intrusive into the deep crust, would have induced poles on its upper surface. But their negative poles for many would be even deeper, so they do not contribute to the total magnetic field. According to this simple reasoning these bodies show polar behavior.

We created an algorithm in MAPLE to perform this study and to calculate the fields and derivatives in the x and z directions in order to form the Euler equations and its corresponding solutions, through the least squares method. From this analysis we obtain the values of \( N=0.5 \) for the dam and \( N=1.0 \) for the failure.

We should clarify that the formulation of programs developed for both the dike failure are unrestricted with the magnetic inclination I, however, the best estimate of the depths will be given for those data that are reduced to the magnetic pole.

3 APPLICATION TO DATA OF NORTHERN PERUVIAN COAST

We used high-resolution magnetometer data of northern peruvian coast, planning for this flight lines 500 m apart in the north-south direction and 1500 m for mooring lines in the east-west direction. The flight altitude was 150m above the surface.

In order to extract two-dimensional profiles three lines were generated whose location is shown in a magnetometer map of the studied area, Figure 2.

After obtaining the magnetic profiles we proceeded to introduce the values of magnetic field and geographic coordinates to a free software called "Euler Deconvolution", in which the data of magnetic inclination for the area of -14.2° and slope of -2.2°, was entered in order to reduce the data to the magnetic pole. In Figure 1a, b, c
this correction can be seen at the top in lighter color. Besides that you can appreciate the vertical and horizontal derivatives of the field, as was necessary to solve the Euler equations.

In order to observe clusters of solutions in certain areas and to associate them with the existing faults in the area put some limits on these solutions with depth. For example it is known that the Paleozoic rocks in the area is at a variable depth of no more than 5 km so, those solutions greater than this value are simply discarded.

With the results obtained by modeling the faults, we find that the optimal value to determine the depth and location for the failures was using a structural index $N = 1.0$. Thus introducing all these values within this program were obtained the solutions shown in Figure 1. For example in Figure 1a, b, c, which corresponds to the profile 1, 2 and 3, you can see a clustering of solutions in the area marked in circles, which gives us an idea that this area could be a rambling a failure at the level of the Paleozoic rocks.
Euler Deconvolution

Figure 1. Magnetometer profiles over the three profiles extracted with the respective estimates of the sources giving rise to the magnetic anomaly. (a) Profile 1, (b) Profile 2 and (c) Profile 3. We used structural index N = 1.0, which according to previous tests corresponds to better position values to determine faults.

In Figure 2 the locations of the three possible failures are plotted according to the results obtained in the interpretation. What we noticed is that the fault deepens more in the West to the South, because the solutions, as we go from profile 1 to 3, are expanded. The other failure, closer to the sea, correspond to a failure shallow (depth less than 2.5 km). This interpretation was corroborated with the ones obtained by in exploration companies through other methods such as seismic.

Figure 2. Solutions to the Euler equations along three profiles, where we denote the presence of two major faults.
4 CONCLUSIONS

We could verify compliance of the homogeneous Euler equation for simple cases of a vertical prism of infinite length, as well as in the case of a failure.

By applying Euler deconvolution of the two simple cases it was noticed that the best values to predict both the depth and position of these simple sources were $N = 0.5$ for the case of infinite vertical prism, while for the fault the position was best predicted with $N = 1.0$. From these values we can conclude that in both cases, the character of the magnetic field strength decreases with distance.

Applying this method to real cases, we see that there is good correspondence between the solutions obtained and those interpreted through other methods such as seismic.

5 REFERENCES


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